Chiral condensate, quark charge and chiral density *

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We study the topological and fermionic vacuum structure of four-dimensional QCD on the lattice by means of correlators of fermionic observables and topological densities. We show the existence of strong local correlations between the topological charge density and the quark condensate, charge and chiral density. By analysis of individual gauge configurations, we visualize that instantons (antiinstantons) carry positive (negative) chirality, whereas the quark charge density fluctuates in sign within instantons.

Over the last two decades several models have been developed to describe the basic properties of QCD, namely quark confinement and chiral symmetry breaking. The most popular are the dual superconductor, leading to confinement, and the instanton liquid model, which explains chiral symmetry breaking and solves the $U_A(1)$ problem [1]. Both models rely on the existence of topological excitations, monopoles and instantons. Instantons have integer topological charge Q which is related to the zero eigenvalues of the fermionic matrix with a gauge field configuration via the Atiyah-Singer index theorem [2]. Apart from this famous connection of topology and fermionic degrees of freedom, here we attempt to systematically shed light on the relationship between the sea-quark distribution and topology [3]. We do this by studying correlators of topological densities with fermionic observables of the form $\bar{\psi}\Gamma\psi$ with $\Gamma = 1, \gamma_4, \gamma_5$. Those quantities are usually referred to as the quark condensate, quark charge density, and the chiral density.

For the implementation of the topological charge on a Euclidian lattice we restrict ourselves to the so-called field theoretic definitions which approximate the topological charge density in the continuum, $q(x) = \frac{g^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\,\Big(F_{\mu\nu}(x)F_{\rho\sigma}(x)\Big)$. We used the plaquette and the hypercube prescription. To get rid of quantum fluctuations and renormalization constants, we employed the Cabibbo-Marinari cooling method. Mathematically and numerically the local quark condensate $\bar{\psi}\psi(x)$ is a diagonal element of the inverse of the fermionic matrix of the QCD action. The other fermionic operators are obtained by inserting the Euclidian γ_4 and γ_5 matrices. We compute correlation functions between two observables $\mathcal{O}_1(x)$ and $\mathcal{O}_2(y)$

$$g(y-x) = \langle \mathcal{O}_1(x)\mathcal{O}_2(y)\rangle - \langle \mathcal{O}_1\rangle\langle \mathcal{O}_2\rangle \qquad (1)$$

and normalize them to the smallest lattice separation $d_{\min}$, $c(y-x) = g(y-x)/g(d_{\min})$. Since topological objects with opposite sign are equally distributed, we correlate the quark condensate with the square of the topological charge density, and similarly for the other quantities.

Our simulations were performed for full SU(3) QCD on an $8^3 \times 4$ lattice with periodic boundary conditions. Applying a standard Metropolis algorithm has the advantage that tunneling between sectors of different topological charges occurs at reasonable rates. Dynamical quarks in Kogut-Susskind discretization with $n_f = 3$ flavors of degenerate mass $m = 0.1$ were taken into account using the pseudofermionic method. We performed runs in the confinement phase at $\beta = 5.2$. Measurements were taken on 2000 configurations separated by 50 sweeps.

Figure 1 shows results for the correlation functions of Eq. (1) with $\mathcal{O}_1$ a local fermionic observable and $\mathcal{O}_2$ a topological density. All correlations exhibit an extension of several lattice spacings. Although cooling of quantum fields is necessary to extract topological structure, the correlations between the topological charge density and both the local chiral condensate and the ab-

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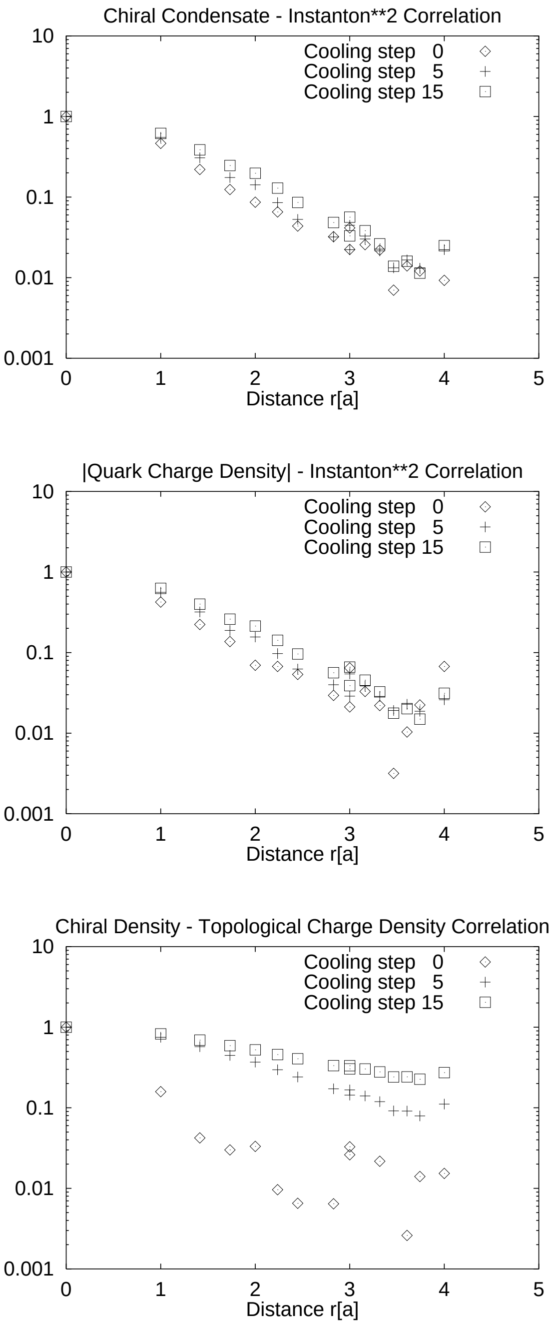

Figure 1. Correlation functions of topological charge density with the quark condensate, quark charge and chiral density shown in the first, second and third row, respectively.

solute value of the quark charge density are nearly cooling-independent. However, cooling (or some other kind of smoothing) is inevitable to obtain nontrivial correlations between the chiral density, $\mathcal{O}_1 = \bar{\psi}\gamma_5\psi(x)$, and the topological charge density. This can be expected since both quantities are correlated via the anomaly.

We now turn to a direct visualization of fermionic densities and topological quantities on individual gauge fields rather than performing gauge averages. We persue this in the following to get insight into the local interplay of topology with the sea-quark distribution. By analyzing dozens of gluon and quark field configurations we obtained the following results. The topological charge is hidden in quantum fluctuations and becomes visible by cooling of the gauge fields. For 0 cooling steps no structure can be seen in $q(x)$, the fermionic observables or the monopole currents, which does not mean the absence of correlations between them. After a few cooling steps clusters of nonzero topological charge density and quark fields are resolved. For more cooling steps both topological charge and quark fields begin to die out and eventually vanish.

In Fig. 2 a typical topologically nontrivial configuration, consisting of an instanton and an antiinstanton, from SU(3) theory with dynamical quarks on the $8^3 \times 4$ lattice in the confinement phase is shown after 15 cooling steps for fixed time slices. We display the positive/negative topological charge density by white/black dots if the absolute value exceeds certain minimal fluctuations. Monopole currents are defined in the maximum Abelian projection and only one type is shown by lines. The upper 3D plot includes the local chiral condensate $\bar{\psi}\psi(x)$, indicated by light grey dots whenever a certain threshold is exceeded. One clearly sees that both the instantons and antiinstantons are surrounded by a cloud of $\bar{\psi}\psi(x) > 0$ [4]. The middle 3D plot exhibits the situation for the quark charge density $\psi^\dagger\psi(x)$ indicated by light and dark grey dots depending on the sign of the net color charge excess. One observes that $\psi^\dagger\psi(x)$ alternates in sign already in one instanton implying trivial correlations $< \psi^\dagger\psi(x)q(y) >= 0$ (not shown in Fig. 1). The lower 3D plot displays the chiral density $\bar{\psi}\gamma_5\psi(x)$ again indicated

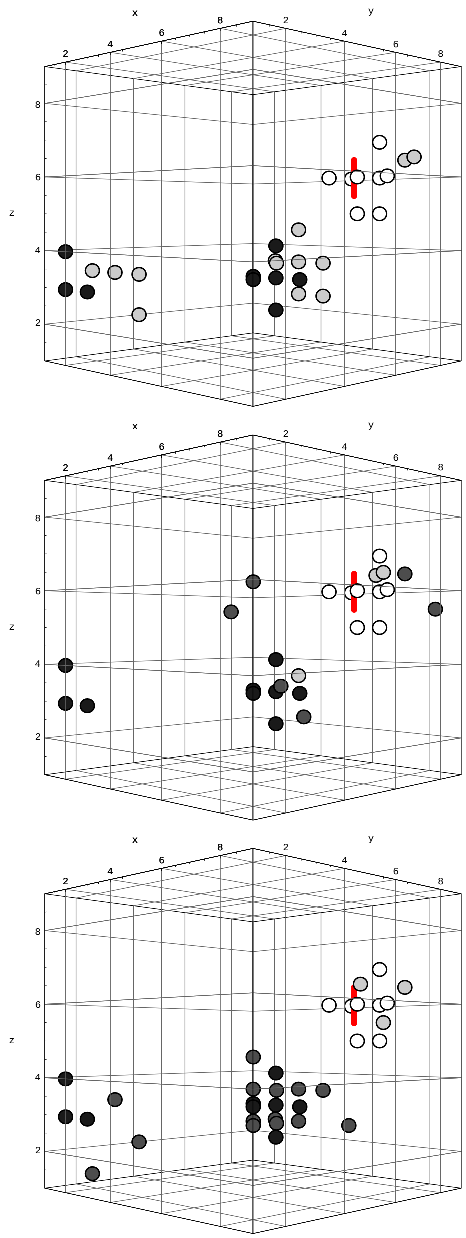

Figure 2. Aspects of the fermionic field for a fixed time slice of a gauge field with an instanton-antiinstanton pair drawn by white and black dots. Upper plot: $\bar{\psi}\psi(x) > 0$ (light grey dots) within the instantons. Middle plot: $\psi^{\dagger}\psi(x)$ alternates in sign (light and dark grey dots) in a single instanton. Lower plot: $\bar{\psi}\gamma_5\psi(x)$ is positive/negative in instanton/antiinstanton.

by light and dark grey dots. One nicely sees that the positive instanton is always surrounded by a lump with $\bar{\psi}\gamma_5\psi(x) > 0$ and vice versa. Combining the above finding of Fig. 1 showing that the correlation functions between fermionic and topological quantities are not very sensitive to cooling together with the 3D images in Fig. 2, we conclude that instantons go hand in hand with clusters of $\bar{\psi}\Gamma\psi(x) \neq 0$, $\Gamma = 1, \gamma_4, \gamma_5$, also in the uncooled QCD vacuum.

In summary, our calculations of correlation functions between topological densities and the fermionic observables yield an exponential decrease. Results for the condensate and the modulus of the quark charge correlators are almost identical, as expected, since the quark condensate reflects the absolute value of the quark charge density. These correlation functions show little cooling dependence.

The correlations unambiguously demonstrate that not only the local chiral condensate but also the quark charge and chiral density take non-vanishing values predominantly in the regions of instantons and monopole loops. Note that for the chiral density this behavior is expected due to the anomaly.

Visualization exhibited that the distribution of sea-quarks is drastically enhanced around centers of nontrivial topology (instantons, monopoles) in Euclidian space-time. It must be emphasized that this represents the situation on a finite lattice with finite quark mass without the extrapolation to the thermodynamic and chiral limit. However, since all such correlators turned out rather independent of the gauge group and choice of the action etc., we expect that they are generic.

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